Higher Resonance Contributions to the Adler–Weisberger Sum Rule in the Large N_c Limit

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Abstract

We determine the N_c–dependence of the resonance contributions to the Adler–Weisberger sum rule for the inverse square $1/g_A^2$ of the axial charge coupling constant and show that in the large N_c limit the contributions of the Roper-like excitations scale as $\mathcal{O}(1/N_c)$. Consistency with the $1/N_c^2$ scaling of the $1/g_A^2$ term in the sum rule requires these contributions to cancel against each other.

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1 Introduction

If the the axial charge commutator in the Adler-Weisberger relation for the axial vector coupling constant is saturated by a discrete set of pion-nucleon resonances (with spin j, parity P and isospin I) the sum rule takes the following form [1], [2]

$$\frac{1}{g_A^2} = 1 + \sum_{(j^P,I)} f_I \frac{g_{jI}^{(\pm)^2}}{g_{\pi NN}^2} m_N^2 \frac{(m_{jI}^2 - m_N^2)^{2j-3}}{(m_N m_{jI})^{2j-1}} \frac{(m_{jI} + \eta\, m_N)^2}{2^{j-\frac{7}{2}}} \frac{[(j+\frac{1}{2})!]^2}{(2j+1)!} \ . \tag{1}$$

In this expression $g_{\pi NN}$ stands for the pseudoscalar pion nucleon coupling constant, $g_{jI}^{(\pm)}$ is the (dimensionless) coupling constant for the spin–j (isospin–I) resonance coupling to the pion-nucleon system in the relativistic Rarita–Schwinger representation [3], and m_N and m_{jI} denote the the nucleon and resonance masses, respectively. Subsequently, f_π denotes the pion weak decay coupling constant.

The quantities g_A, m_N and m_{jI} scale with the number of color degrees of freedom [4, 5] as $\mathcal{O}(N_c)$, whereas $f_\pi \sim \mathcal{O}(\sqrt{N_c})$ and $g_{\pi NN} \sim \mathcal{O}(N_c^{\frac{3}{2}})$. The mass differences $m_{jI} - m_N$ are either of order $\mathcal{O}(N_c^0)$ or of order $\mathcal{O}(1/N_c)$. The former scaling behavior is the usual situation for resonances in meson-baryon scattering [5] and applies e.g. for the Roper-like resonances as well as for the odd-parity states. The latter scaling behavior is special for e.g. the $\Delta(1232)$ and its partners in the $(j^+, I(j))$ tower that belongs to the contracted SU(4)-symmetry, which applies for the baryon states in strong coupling models [6, 7] and the $1/N_c$ expansion [8, 9]. The isospin factor f_I takes the value 1 for the $I = \frac{1}{2}$ resonances and the value $-\frac{1}{3}$ for the $I = \frac{3}{2}$ ones. (Note that resonances with $I > \frac{3}{2}$ cannot couple to the pion-nucleon system). The parameter η in (1) takes the values $\pm$ and corresponds to the $\pm$ structure in the exponent of the parity $P = (-1)^{j\pm 1/2}$ of the intermediate resonance. For example, the sign "+" corresponds to the odd-parity resonances (with $(-1)^{j+\frac{1}{2}} = -1$) as well as for the Δ–resonance and its Roper like excitation, whereas the "−" sign has to be used for the Nucleon-Roper, the Delta-odd-parity resonant excitations etc.

With the exception of the coupling constants $g_{jI}^{(\pm)}$ the scaling with N_c of all the quantities that appear in eq.(1) is known. The left hand side of eq.(1) is of order $\mathcal{O}(1/N_c^2)$. The right hand side, however, because of its first summand, has an apparent order $\mathcal{O}(N_c^0)$ term. The resonance contributions on the right hand side of eq.(1) therefore have to combine so as to (a) exactly cancel the first summand

(1) and to (b) give the required $\mathcal{O}(1/N_c^2)$ behavior. This requirement was used in ref.[10] to get the N_c scaling of the coupling constants $g_{jI}^{(\pm)}$. We here show that the quark model implies that the coupling constants for the Roper-like resonances scale as N_c and not as $N_c^{3/2}$ as inferred from the sum rule in ref.[10]. This N_c dependence implies that these resonances give contributions of order $1/N_c$ to the sum rule, and thus consistency with the r.h.s. requires these contributions to cancel against each other. That cancellation is made possible by the opposite signs of the contributions from the Roper and Δ-Roper resonances in the sum rule.

2 N_c scaling of the coupling constants

In the relativistic Rarita-Schwinger representation πN-resonance vertices are [3]:

$$\langle N(p)|\chi^\beta|j(p+q)\rangle = \frac{g_{jI}^{(+)}}{m_N^{j-\frac{1}{2}}}\bar{u}(p)u_{\mu_1\cdots\mu_{j-\frac{1}{2}}}(p+q)\ q'^{\mu_1}\cdots q'^{\mu_{j-\frac{1}{2}}}\ \xi_{\frac{1}{2}}^+\left\{\begin{array}{c}\tau^\beta\xi_{\frac{1}{2}}\\[6pt]\xi_{\frac{3}{2}}^\beta\end{array}\right\},\quad (2)$$

$$\langle N(p)|\chi^\beta|j(p+q)\rangle = \frac{ig_{jI}^{(-)}}{m_N^{j-\frac{1}{2}}}\bar{u}(p)\gamma_5 u_{\mu_1\cdots\mu_{j-\frac{1}{2}}}(p+q)\ q'^{\mu_1}\cdots q'^{\mu_{j-\frac{1}{2}}}\ \xi_{\frac{1}{2}}^+\left\{\begin{array}{c}\tau^\beta\xi_{\frac{1}{2}}\\[6pt]\xi_{\frac{3}{2}}^\beta\end{array}\right\},(3)$$

where $u(p)$ and $u_{\mu_1\cdots\mu_{j-\frac{1}{2}}}(p+q)$ are in turn the spinor and and totally symmetric tensor-spinor wave functions that describe the proton and the spin–j resonance, $\xi_{\frac{1}{2}}$, $\xi_{\frac{3}{2}}^\beta$ are the wave functions of the isospin–$\frac{1}{2}$ or $\frac{3}{2}$ resonances, respectively, and $\xi_{\frac{1}{2}}^+$ is the isospin–$\frac{1}{2}$ wave function of the proton. The two equations correspond to resonances of parity $(-1)^{j+\frac{1}{2}}$ and $(-)^{j-\frac{1}{2}}$, respectively. The four-momentum q'_μ is defined as

$$q'_\mu = q_\mu - \frac{q(p+q)}{m_{jI}^2}(p+q)_\mu\ .\tag{4}$$

Compare now the N_c behavior of the non-relativistic reduction (in the baryon degrees of freedom) of eqs.(2) and (3) with the results for the non-relativistic quark model in the large N_c limit. In this limit the non-relativistic reduction is well justified, since the baryon masses scale as $\mathcal{O}(N_c)$ and can be made arbitrarily heavy. The naive quark model can also be relied upon in this limit as we only need the leading N_c behavior of the resonance couplings, which is model-independent [8]. In the non-relativistic limit the *spin-isospin*-structure of the one-body vertex operators

3

sandwiched between the proton state $|N\rangle$ and a (by the transition allowed) baryon state $|B_j\rangle$ can take only one of the following four forms: a scalar-isoscalar sum

$$\langle N|\frac{G}{f_\pi}\sum_{\nu=1}^{N_c}\mathbf{1}_\nu|B_j\rangle \sim \begin{cases} \mathcal{O}(\sqrt{N_c}) \\ \mathcal{O}(N_c^0) \end{cases}, \tag{5}$$

a pseudoscalar-isovector sum

$$\langle N|\frac{G}{f_\pi}\sum_{\nu=1}^{N_c}\vec{\sigma}_\nu\cdot\vec{q}\,\tau_\nu^\beta|B_j\rangle \sim \begin{cases} \mathcal{O}(\sqrt{N_c}) \\ \mathcal{O}(N_c^0) \end{cases}, \tag{6}$$

a scalar-isovector sum

$$\langle N|\frac{G}{f_\pi}\sum_{\nu=1}^{N_c}\tau_\nu^\beta|B_j\rangle \sim \begin{cases} \mathcal{O}(1/\sqrt{N_c}) \\ \mathcal{O}(1/N_c) \end{cases}, \tag{7}$$

or a pseudoscalar-isoscalar sum

$$\langle N|\frac{G}{f_\pi}\sum_{\nu=1}^{N_c}\vec{\sigma}_\nu\cdot\vec{q}\,|B_j\rangle \sim \begin{cases} \mathcal{O}(1/\sqrt{N_c}) \\ \mathcal{O}(1/N_c) \end{cases}. \tag{8}$$

Here the index ν counts the N_c different quarks of the proton and the spin–j resonance. In (8) G is a pion-quark coupling strength which scales as $\mathcal{O}(N_c^0)$. The pion decay constant f_π appears through the normalization of the pion field in the vertex operator. The spatial vector $\vec{q}$ is the pion three-momentum in the resonance rest frame which also scales as $\mathcal{O}(N_c^0)$ for fixed kinematics. In the equations above the upper scaling behavior on the r.h.s. applies to the case when B_j is the nucleon, or a state in which all quarks are in their spatial ground state (e.g. the lowest Δ resonance). The lower scaling behavior applies for such baryon states with one or two quarks in an excited spatial state (e.g. the Nucleon- or Δ-Roper). In that case the symmetrization of the color-singlet baryon state introduces an additional normalization factor $1/\sqrt{N_c}$ [11], which leads to a similar extra factor in the N_c dependence of the matrix element. This is consistent with the fact that – because of unitarity – the $\pi N \to \pi N$ scattering amplitude can at most scale as $\mathcal{O}(N_c^0)$: The direct Born diagram with a nucleon as intermediate state scales by itself as $\mathcal{O}(N_c)$ since both vertices scale as $\mathcal{O}(\sqrt{N_c})$. In ref.[8] it was shown that this leading N_c scaling cancels

4

when the crossed Born diagram *and* the corresponding direct and crossed diagrams with the (in the large N_c limit degenerate $SU(4)$ partner) Δ are included. However, in case the intermediate state in the $\pi N \to \pi N$ scattering is a (with the nucleon non-degenerate) baryon-resonance, the direct Born diagram cannot be cancelled any longer in this way. Thus, in order to be consistent with unitarity, the vertex itself can at most scale as $\mathcal{O}(N_c^0)$ in agreement with the scaling behavior derived from the normalization of the excited baryon state [11].

Note that in the vertex sum (6) the single quark $\vec{\sigma}_\nu \cdot \vec{q} \tau_\nu^\beta$ contributions add up coherently and lead to the the same N_c scaling as in eq.(5), whereas the other two vertex sums (7) and (8) are suppressed by $1/N_c$ since the quark contributions only add up destructively. In fact only the vertex sums (6) and (7) contribute here because of the isovector nature of the pion coupling. The N_c behavior of the pion-proton vertices in the naive quark model is thus determined.

The non-relativistic reduction of the Rarita-Schwinger-type vertex matrix elements (2) and (3) is standard. Using the *totally symmetric* nature of the j-$\frac{1}{2}$ tensor spinors one can identify – in the non-relativistic limit and in spin-isospin space – the matrix elements (2) with the pseudoscalar-isovector structure, eq.(6), of the naive quark model, if j-$\frac{1}{2}$ is odd, and with the scalar-isovector one, eq.(7), if j-$\frac{1}{2}$ is even. For the vertex matrix element (3) the fact has to be taken into account that the γ_5 Dirac matrix links one upper and one lower component of the proton and the spin–j resonance and therefore induces an extra factor $\vec{\sigma} \cdot (\vec{p} + \vec{q})/2m_{jI} - \vec{\sigma} \cdot \vec{p}/2m_N = \vec{\sigma} \cdot \vec{q}/2m_N + \mathcal{O}(1/N_c^2)$ [#1]. Therefore the vertex matrix element (3) leads to the pseudoscalar-isovector structure, eq.(6), for j-$\frac{1}{2}$ even and to the scalar-isovector one, eq.(7), for j-$\frac{1}{2}$ odd. Thus, the N_c behavior of the $g_{jI}^{(\pm)}$'s can now be predicted.

The scaling of the Roper resonance coupling $g_{\frac{1}{2}\frac{1}{2}}^{(-)}$ [#2] follows via eqs.(3) and (6) from

$$\frac{g_{\frac{1}{2}\frac{1}{2}}^{(-)}}{2m_N} \sim \frac{G}{f_\pi \sqrt{N_c}} \times N_c \sim \mathcal{O}(1) \tag{9}$$

and is thus $\mathcal{O}(N_c)$. The scaling of the odd-parity-nucleon-like coupling $g_{\frac{1}{2}\frac{1}{2}}^{(+)}$ results

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[#1]Note that in the rest frame of the resonance $\vec{q} = \vec{q}\,'$.

[#2]The "$-$" sign in the notation of ref.[3] is reflecting the "$-$" sign in $j - \frac{1}{2}$ and should not be mixed up with the parity of the Roper state which of course is positive.

from

$$g^{(+)}_{\frac{1}{2}\frac{1}{2}} \sim \frac{G}{f_\pi \sqrt{N_c}} \sim \mathcal{O}(1/N_c) \tag{10}$$

(see eqs.(2) and (7)) and is $\mathcal{O}(1/N_c)$. The scaling behavior of the Δ (ground state) coupling $g^{(+)}_{\frac{3}{2}\frac{3}{2}}$ can be deduced from

$$\frac{g^{(+)}_{\frac{3}{2}\frac{3}{2}}}{m_N} \sim \frac{G}{f_\pi} N_c \sim \mathcal{O}(\sqrt{N_c}) \tag{11}$$

as $N_c^{\frac{3}{2}}$, whereas the coupling $g^{(+)}_{\frac{3}{2}'\frac{3}{2}}$ of the Roper-like Δ scales – because of the $1/\sqrt{N_c}$ normalization – only as N_c:

$$\frac{g^{(+)}_{\frac{3}{2}'\frac{3}{2}}}{m_N} \sim \frac{G}{f_\pi \sqrt{N_c}} \times N_c \sim \mathcal{O}(N_c^0) \tag{12}$$

(see eqs.(2) and (6)). The coupling constant $g^{(-)}_{\frac{3}{2}\frac{3}{2}}$ of the odd-parity Δ–resonance also behaves as N_c (see eqs.(3) and (7)), since

$$\frac{g^{(-)}_{\frac{3}{2}\frac{3}{2}}}{m_N^2} \sim \frac{G}{f_\pi \sqrt{N_c}} \sim \mathcal{O}(1/N_c) \ . \tag{13}$$

In general, comparing the non-relativistic reduction of eqs.(2) or (3) with the N_c-counting of eqs.(6) or (7) one can derive the following relations for the coupling constants of the $(j^\pm, I)$ states with $I=\frac{1}{2}$ or $\frac{3}{2}$: [#3]

$$
\begin{aligned}
g^{(+)}_{jI} &= \mathcal{O}(N_c^{j-\frac{1}{2}}) \quad \text{for} \quad P = (-1)^{j+\frac{1}{2}} = + \\
g^{(+)}_{jI} &= \mathcal{O}(N_c^{j-\frac{3}{2}}) \quad \text{for} \quad P = (-1)^{j+\frac{1}{2}} = - \\
g^{(-)}_{jI} &= \mathcal{O}(N_c^{j+\frac{1}{2}}) \quad \text{for} \quad P = (-1)^{j-\frac{1}{2}} = + \\
g^{(-)}_{jI} &= \mathcal{O}(N_c^{j-\frac{1}{2}}) \quad \text{for} \quad P = (-1)^{j-\frac{1}{2}} = - \ ,
\end{aligned}
\tag{14}
$$

(see the 5th row in Table 1).

3 N_c scaling of the resonance contributions to the sum rule

The N_c scaling of all the quantities that enter on the right hand side of the Adler-Weisberger sum rule (1) is now determined. In the 7th row of Table 1 the contri-

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[#3]With the exception of the lowest Δ state for which the coupling constant behave as $\mathcal{O}(N_c^{\frac{3}{2}})$ and not as $\mathcal{O}(N_c)$.

butions of the single resonances with $j \leq 7/2$ are listed. Using the results of Table 1 one finds that (a) the $\Delta(1232)$ contributes to leading order $\mathcal{O}(N_c^0)$ to the r.h.s. of the sum rule, (b) the N-Roper, the Δ-Roper, the $(\frac{3}{2}^+, \frac{1}{2})$, the $(\frac{5}{2}^+, \frac{1}{2})$, the $(\frac{1}{2}^+, \frac{3}{2})$ etc. contribute to next-to-leading order, $\mathcal{O}(1/N_c)$, and (c) the contribution of e.g. the odd-parity $(\frac{1}{2}^-, \frac{1}{2})$ nucleon excitation is suppressed by $\mathcal{O}(1/N_c^3)$.

Since the contributions of the the radial excitations of the nucleon- and the Δ-resonances contribute to next-to-leading order, $\mathcal{O}(1/N_c)$, they have to cancel in order to keep consistency with the scaling of the l.h.s. of the sum rule. The contributions from these states therefore have to be considered with some care when the Adler-Weisberger sum-rule is applied to large N_c models as the Skyrme-model (or its extensions) or large N_c quark models. The Roper-like states, on the one side, and the $\Delta(1232)$ resonance, on the other side, contribute for rather different reasons significantly to the r.h.s. of the sum rule in eq.(1). In the large N_c limit the latter is degenerate with the nucleon (the mass splitting goes as $\mathcal{O}(1/N_c)$) and has a width that vanishes with $1/N_c^2$. The Ropers, on the other hand, do not become degenerate with the nucleon in the large N_c limit as the splitting is $\mathcal{O}(N_c^0)$, but their widths are also of order $\mathcal{O}(N_c^0)$ and therefore large enough to leave strength left at the nucleon pole. The masses of the other resonances are order $\mathcal{O}(N_c^0)$ above the nucleon pole and have widths that are only of order $\mathcal{O}(1/N_c^2)$. Therefore they have no strength at the nucleon pole in the large N_c limit and may be neglected.

The widths of the spin–j isospin–I resonances are given by the expression [1, 3]:

$$\Gamma_{jI}^{(\pm)} = 3|f_I| \frac{g_{jI}^{(\pm)^2}}{4\pi m_N^{2j-1}} \frac{2^{j-\frac{1}{2}}[(j-\frac{1}{2})!]^2}{(2j)!} \frac{E_N + \eta\, m_N}{m_{jI}} (\vec{p}^{\,2})^j \ . \tag{15}$$

Here $\vec{p}$ and E_N are the three-momentum and the energy of the nucleon in the j-resonance rest frame, respectively. For fixed kinematics E_N scales as $\mathcal{O}(N_c)$, whereas $|\vec{p}|$ normally scales as $\mathcal{O}(N_c^0)$. Note, however, that the nucleon kinetic energy $E_N - m_N$ scales as $\mathcal{O}(1/N_c)$, which is obvious in the non-relativistic limit. Furthermore for the $\Delta(1232)$ (and its partners in the (j^+, j) tower) the three-momentum $\vec{p}$ does not scale as $\mathcal{O}(N_c^0)$, but rather as $1/N_c$. From the expression (15), it then follows that the widths of all the Roper like excitations scale as $\mathcal{O}(N_c^0)$, whereas those of the odd-parity resonances scale as $\mathcal{O}(1/N_c^2)$ (see Table 1). Finally, as pointed out above, the width of the $\Delta(1232)$ falls with N_c at least as $\mathcal{O}(1/N_c^2)$.

4 Conclusions

The contribution C_Δ of the Δ resonance to the right hand side of the Adler–Weisberger sum rule (1) is:

$$C_\Delta = -\frac{2}{9} \frac{g_{\frac{3}{2}\frac{3}{2}}^{(+)2}}{g_{\pi NN}^2} \frac{(m_\Delta + m_N)^2}{m_\Delta^2} . \tag{16}$$

In the large N_c limit when $m_\Delta \to m_N$ one finds that $C_\Delta \to -1$ [2, 12] in accordance with the prediction of the contracted SU(4) spin–flavor symmetry. The contribution C_Δ alone is therefore sufficient to cancel the terms of order N_c^0 in the sum rule eq.(1) exactly. However, as shown in section 3, the contributions from the Roper–like, as well as the D_{13} negative parity resonances in the large N_c limit scale as $1/N_c$ and consistency therefore requires these to cancel against each other [#4]. This behavior of the Roper-like states is in line with the unitarity limit, $\mathcal{O}(N_c^0)$, of the total cross sections $\sigma_{\text{tot}}^{\pi^\pm p}$ in $\pi^\pm$-proton scattering which appear on the r.h.s. of the integral form of the Adler-Weisberger relation

$$\frac{1}{g_A^2} = 1 + \frac{2m_N^2}{\pi g_{\pi NN}^2} \int_{m_\pi}^\infty \frac{d\nu}{\nu} \sqrt{\nu^2 - m_\pi^2} \left(\sigma_{\text{tot}}^{\pi^- p}(\nu) - \sigma_{\text{tot}}^{\pi^+ p}(\nu) \right), \tag{17}$$

($\nu = p \cdot q/m_N$). Note that the $I = \frac{1}{2}$ resonances can only show up in the $\pi^- p$ scattering. We have shown – using unitarity – that the corresponding resonance cross sections can at most scale as $\mathcal{O}(N_c^0)$ and that in fact for the Roper-like resonances the upper bound is saturated. Taking into account the prefactor of the integral one easily finds that the total contribution of the Roper-like states scales as $\mathcal{O}(1/N_c)$ in agreement with our derivation presented above. In the case of the $I = \frac{3}{2}$ resonances both $\sigma_{\text{tot}}^{\pi^\pm p}$ cross sections contribute. The net result of the $I = \frac{3}{2}$ Roper-like resonances is still of order $\mathcal{O}(1/N_c)$ and negative in order to remove the positive $\mathcal{O}(1/N_c)$ contribution of their corresponding $I = \frac{1}{2}$ partners. For the non-resonant $\pi^\pm$-proton scattering, however, it can be shown [15] that the cross sections cancel to order $\mathcal{O}(1/N_c)$ since the π^- meson is counting the $(N_c + 1)/2$ u quarks of the proton, whereas the π^+ is counting the respective $N_c/2$ d quarks. The total contribution of the non-resonant π-proton scattering to the Adler-Weisberger integral (17) is therefore of order $\mathcal{O}(1/N_c^2)$ [15]. The contributions of the S_{11} and D_{15}

[#4]The importance of the Nucleon-Roper resonance for the saturation of the chiral axial charge commutator has already been emphasized in [14].

odd-parity resonances and the other resonances whose decay width are of the order $\mathcal{O}(1/N_c^2)$ appear suppressed by the order of $\mathcal{O}(1/N_c^3)$ as compared to the leading and next-to-leading effects discussed above. The predicted behavior is qualitatively in line with the one extracted from the data on the πN decay widths of the $j < \frac{7}{2}$ resonances below $2.2 \; GeV$.

Acknowledgements

References

[1] A. Della Selva and L. Masperi, Nuovo Cimento **50 A** (1967) 5393.

[2] F.D. Mazzitelli, L. Masperi, Phys. Rev. **D35** (1987) 368.

[3] V. De Alfaro, S. Fubini, G. Furlan, C. Rossetti, "Currents in Hadron Physics", (North-Holland, Amsterdam) 1973.

[4] G. 't Hooft, Nucl. Phys. **B72** (1974) 461.

[5] E. Witten, Nucl. Phys. **B160** (1979) 57.

[6] K. Bardakci, Nucl. Phys. **B243** (1984) 197.

[7] J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52** (1984) 87.

[8] R. Dashen and A.V. Manohar, Phys. Lett. **B315** (1993) 425, 438.

[9] E. Jenkins, Phys. Lett. **B315** (1993) 431, 441, 447.

[10] A. Kakuto and F. Toyoda, Prog. Theor. Phys. **66** (1981) 2307.

[11] S. Coleman, "1/N" (1979) in "Aspects of Symmetry", Selected Erice Lectures, (Cambridge University Press, Cambridge) 1985.

[12] H. Yamagishi and I. Zahed, Mod. Phys. Lett. **A7** (1992) 1105.

[13] Particle Data Group, *Part II: Review of Particle Properties*, Phys. Rev. **D45** (1992) No. 11-II.

[14] M. Kirchbach and D.O. Riska, Nuovo Cimento **104** (1991) 1837.

[15] W. Broniowski, private communication.

I	j	P	resonance	$g_{jI}^{(\pm)}$	η	R.H.S. of (1)	data	$\Gamma_{jI}^{(\pm)}$
$\frac{1}{2}$	$\frac{1}{2}$	$+$	$N(1440)P_{11}$	N_c	$(-)$	$1/N_c$	0.147	1
	$\frac{1}{2}$	$+$	$N(1710)P'_{11}$	N_c	$(-)$	$1/N_c$	0.003	1
	$\frac{1}{2}$	$-$	$N(1535)S_{11}$	N_c^{-1}	$(+)$	$1/N_c^3$	0.025	$1/N_c^2$
	$\frac{1}{2}$	$-$	$N(1650)S'_{11}$	N_c^{-1}	$(+)$	$1/N_c^3$	0.025	$1/N_c^2$
	$\frac{3}{2}$	$+$	$N(1720)P_{13}$	N_c	$(+)$	$1/N_c$	0.009	1
	$\frac{3}{2}$	$-$	$N(1520)D_{13}$	N_c	$(-)$	$1/N_c^3$	0.060	$1/N_c^2$
	$\frac{3}{2}$	$-$	$N(1700)D'_{13}$	N_c	$(-)$	$1/N_c^3$	0.004	$1/N_c^2$
	$\frac{5}{2}$	$+$	$N(1680)F_{15}$	N_c^3	$(-)$	$1/N_c$	0.061	1
	$\frac{5}{2}$	$-$	$N(1675)D_{15}$	N_c	$(+)$	$1/N_c^3$	0.050	$1/N_c^2$
	$\frac{7}{2}$	$-$	$N(2190)G_{17}$	N_c^3	$(-)$	$1/N_c^3$	0.016	$1/N_c^2$
$\frac{3}{2}$	$\frac{1}{2}$	$+$	$\Delta(1910)P_{31}$	N_c	$(-)$	$1/N_c$	-0.006	1
	$\frac{1}{2}$	$-$	$\Delta(1620)S_{31}$	N_c^{-1}	$(+)$	$1/N_c^3$	-0.010	$1/N_c^2$
	$\frac{1}{2}$	$-$	$\Delta(1900)S'_{31}$	N_c^{-1}	$(+)$	$1/N_c^3$	-0.004	$1/N_c^2$
	$\frac{3}{2}$	$+$	$\Delta(1232)P_{33}$	$N_c^{\frac{3}{2}}$	$(+)$	1	-0.772	$1/N_c^2$
	$\frac{3}{2}$	$+$	$\Delta(1600)P'_{33}$	N_c	$(+)$	$1/N_c$	-0.037	1
	$\frac{3}{2}$	$+$	$\Delta(1920)P''_{33}$	N_c	$(+)$	$1/N_c$	-0.005	1
	$\frac{3}{2}$	$-$	$\Delta(1700)D_{33}$	N_c	$(-)$	$1/N_c^3$	-0.019	$1/N_c^2$
	$\frac{5}{2}$	$+$	$\Delta(1905)F_{35}$	N_c^3	$(-)$	$1/N_c$	-0.012	1
	$\frac{5}{2}$	$-$	$\Delta(1930)D_{35}$	N_c	$(+)$	$1/N_c^3$	-0.017	$1/N_c^2$
	$\frac{7}{2}$	$+$	$\Delta(1950)F_{37}$	N_c^3	$(+)$	$1/N_c$	-0.047	1

Table 1: The N_c scaling behavior of the dimensionless coupling constants $g_{jI}^{(\pm)}$, the widths $\Gamma_{jI}^{(\pm)}$ of the $(j^\pm, \frac{1}{2})$ and $(j^\pm, \frac{3}{2})$ resonances (with $j \leq \frac{7}{2}$), and their contributions to the right hand side of the Adler-Weisberger sum rule from eq. (1). The column denoted "data" contains the contributions of the single resonances to the Adler–Weisberger sum rule as obtained in exploiting the experimental values on the πN partial decay widths of the resonances. For notations and resonance data see ref.[13] .